4. Pipe manufacturing—Pipes can be manufactured in many ways. Pipes can be manufactured by casting, drawing, bending of sheets, welding, riveting. A pipe with a line of joint is known as seam pipe. A pipe without line of joint is known as seamless pipe.

5. Materials of pipe—Pipes are usually made of cast iron, wrought iron, steel, copper, aluminium, concrete, copper, asbestos, rubber, P.V.C (polyvinyl chloride) etc.

6. Pipe fittings—There are different types of pipe fittings. These are employed for various functions in a pipe line. Various pipe fittings and their point of applications are shown in fig. 8·1.

8·2. Definitions Regarding Pipe Flow

1. Wetted Perimeter—Length of cross-section of a pipe or channel in contact with liquid is called wetted perimeter. It is denoted by P. S. I. unit of wetted perimeter is metre.

2. Wetted Area—Area of cross-section of a pipe or channel in contact with liquid. It is denoted by A.

3. Hydraulic Mean Depth or Hydraulic Radius—It is ratio of wetted area to its wetted perimeter. It is denoted by m. Therefore, hydraulic mean depth,

$$\boxed{m = \frac{A}{P}}$$

(a) Circular pipe running full—Consider fig. 8·2 (a).

Wetted area, $A = \frac{\pi}{4} d^2$

Wetted perimeter, $P = \pi d$

∴ hydraulic mean depth, $m = \frac{A}{P} = \frac{\frac{\pi}{4} d^2}{\pi d}$

so, $m = \frac{d}{4}$

(b) Circular pipe running partially filled—Let r be the radius of the pipe. Consider fig. 8·2 (b).

Wetted area, $A = \frac{r^2}{2}(\theta - \sin\theta)$

Wetted perimeter, $P = r\theta$

∴ $m = \frac{A}{P} = \frac{\frac{r^2}{2}(\theta - \sin\theta)}{r\theta}$

so, $m = \frac{r}{2}\left(1 - \frac{\sin\theta}{\theta}\right)$

(c) Rectangular channel—Consider fig. 8·2 (c).

Wetted area, $A = bh$

Wetted perimeter, $P = b + 2h$

∴ $m = \frac{A}{P} = \frac{bh}{b+2h}$

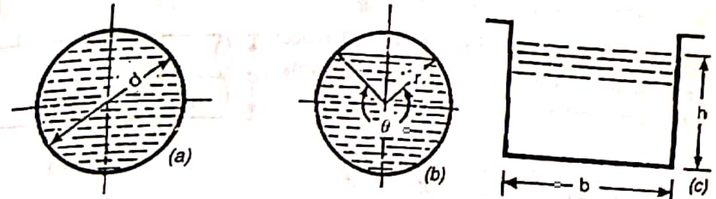


Fig. 8.2 (a, b, c)

3. Hydraulic Slope or Hydraulic Gradient—It is usually denoted by i. Hydraulic slope is the ratio of head loss due to friction h_f in the pipe flow to the length of pipe. Thus,

$$\text{hydraulic slope} = \frac{\text{head loss due to friction}}{\text{length of the pipe}}$$

or $$i = \frac{h_f}{l}$$

8·3. Froude's Experiment

When liquid flows through a pipe, flow is resisted by rough surface of the pipe which offers frictional resistance. There is loss of energy of the liquid. It is known as ***frictional head loss***. It is denoted by h_f.

Actually head loss of the liquid is due to frictional loss between layers of the viscous liquid. Velocity of liquid is zero at the surface of pipe and increases to maximum at the centre of pipe. Variation of velocity of flow is parabolic. So, it can be concluded here that frictional loss is by virtue of ***viscous resistance***.

Froude conducted series of experiments to determine frictional resistance offered to flowing water. He arrived at following conclusions—

(i) Frictional resistance is directly proportional to the square of the mean velocity of flow.
(ii) Frictional resistance per unit area of the surface is constant for long lengths.
(iii) Frictional resistance depends on nature of the surface.

Therefore, frictional resistance,

$$\boxed{R \propto AV^2}$$

or $R = f' AV^2$

where f' = frictional resistance per unit wetted area per unit velocity (Froude's frictional coefficient [ML^{-3}])

A = area of wetted cross-section

and V = mean velocity of flow

8·4. Head Loss in Pipes Due To Friction (Darcy – Weisbach Equation)

without proof

Consider a horizontal pipe of diameter d running full of liquid of density ρ. Let V be the mean velocity of the flow. Refer Fig. 8.3.

Two piezometer tube are fixed l distance apart at J and K.

Pressure heads at these points of the pipe are $\frac{p_1}{\rho g}$ and $\frac{p_2}{\rho g}$.

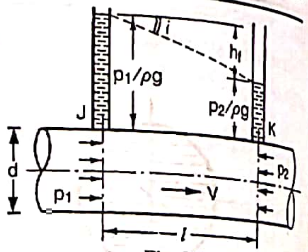


Fig. 8·3

Loss of energy takes place between J and K, so difference between pressure heads at two points will be h_f. Now applying Bernoulli's theorem between J and K,

$$\frac{p_1}{\rho g}+\frac{V^2}{2g}+Z = \frac{p_2}{\rho g}+\frac{V^2}{2g}+Z+h_f$$

Velocity V is same because cross-section of pipe is uniform and height Z is also same because pipe is horizontal. So,

$$\frac{p_1}{\rho g}-\frac{p_2}{\rho g} = h_f \quad ...(1)$$

We know from Froude's experiment that,

frictional resistance = friction resistance per unit wetted area per unit velocity × wetted area × (velocity)^2

or $$R = f' \times \pi dl \times V^2 \quad ...(2)$$

If A be the area of cross-section of the pipe then net force responsible for flow of liquid between J and K

$$F = (p_1 - p_2)\, A = (p_1 - p_2)\frac{\pi d^2}{4} \quad ...(3)$$

As liquid flows without acceleration so two forces should be in equilibrium, thus

$$F = R$$

equating RHS of eq. (2) and (3),

$$(p_1 - p_2)\frac{\pi d^2}{4} = f' \times \pi dl \times V^2$$

only theory

or $$p_1 - p_2 = f' \times \frac{4lV^2}{d}$$

or $$\frac{p_1}{\rho g}-\frac{p_2}{\rho g} = \frac{f'}{\rho g}\times\frac{4lV^2}{d}$$

from eq. (1), $$h_f = \frac{f'}{\rho g}\times\frac{4lV^2}{d}$$

Putting $$\frac{f'}{\rho g} = \frac{f}{2g}$$

Where f is coefficient of friction or frictional factor, which is a dimensionless quantity.

So, $$h_f = \frac{f}{2g}\times\frac{4lV^2}{d}$$

∴ head loss due to friction, $$\boxed{h_f = \frac{4flV^2}{2gd}} \quad ...(4)$$

Note—1. Sometimes value of 4f is given instead of f. Therefore, students are advised to take care.
2. Coefficient of friction and Reynold number are related as follows :

(a) $$f = \frac{16}{R_N} \quad \text{(for } R_N \text{ upto 2000)}$$

(b) $$f = \frac{0.0791}{R_N^{1/4}} \quad \text{(for } R_N = 4000 \text{ to } 10^5)$$

(c) $$f = 0.0008 + \frac{0.05525}{R_N^{0.237}} \quad \text{(for } R_N > 10^5)$$

8·5. Chezy's Formula

only theory (without proof)

Chezy's formula of fluid flow may be used to determine head loss due to friction.

We have derived that head loss due to friction,

$$h_f = \frac{4flV^2}{2gd}$$

But, $$V^2 = \frac{h_f}{l}\cdot\frac{d}{4}\cdot\frac{2g}{f} \quad ...(1)$$

But $$\frac{h_f}{l} = i = \text{hydraulic gradient}$$

$$\frac{d}{4} = m = \frac{A}{P} = \text{hydraulic mean radius}$$

∴ from (1), $$V^2 = m.i.\frac{2g}{f}$$

or $$V = \sqrt{\left(\frac{2g}{f}m.i\right)}$$

Chezy's formula, $$\boxed{V = C\sqrt{(mi)}} \quad .....(2)$$

where $\sqrt{\left(\frac{2g}{f}\right)} = C$ = **Chezy's constant**

Note that Chezy's constant depends upon coefficient of friction f. Following expression is suggested for determining f.

- For new smooth pipes $\boxed{f = 0{\cdot}005\left(1+\frac{1}{35d}\right)} \approx 0{\cdot}005$
- For old rough pipes $\boxed{f = 0{\cdot}01\left(1+\frac{1}{35d}\right)} \approx 0{\cdot}01$

d is diameter of pipe in metre (m).

Example 1 : A 200 m long horizontal pipe of 20 cm diameter is running full of water at 3 m/s velocity. Determine head loss due to friction, hydraulic mean depth and hydraulic slope if f = 0·01 in Darcy's equation. If C = 59 then find head loss by Chezy's formula also. **(UPBTE)**

Solution : Head loss due to friction by Darcy's equation,

$$h_f = \frac{4fLV^2}{2gd}$$

where $f = 0{\cdot}01$, $l = 200$ m, $V = 3$ m/s, $d = 20$ cm $= 0{\cdot}2$ m

$$\therefore \quad h_f = \frac{4 \times 0{\cdot}01 \times 200 \times 3^2}{2 \times 9{\cdot}81 \times 0{\cdot}2} = \mathbf{18{\cdot}35\ m} \quad \textbf{Ans.}$$

Hydraulic mean depth, $m = \frac{A}{P}$

Wetted area,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 0{\cdot}2^2 = 0{\cdot}031 \ \text{m}^2$$

Wetted perimeter, $P = \pi d = \pi \times 0{\cdot}2$ m

$$\therefore \quad m = \frac{0{\cdot}0314}{\pi \times 0{\cdot}2} = \mathbf{0{\cdot}05\ m} \quad \textbf{Ans.}$$

Hydraulic slope, $i = \frac{h_f}{l} = \frac{18{\cdot}35}{200} = \mathbf{0{\cdot}09175}$ **Ans.**

Chezy's formula, $V = C\sqrt{(m.i)}$

But $V = 3$ m/s, $C = 59$, $i = \frac{h_f}{l}$, $m = 0{\cdot}05$ m

$$\therefore \quad 3 = 59\sqrt{\left(0{\cdot}05 \times \frac{h_f}{l}\right)} \quad \text{or} \quad 9 = 59^2 \times 0{\cdot}05 \times \frac{h_f}{l}$$

Hence,

$$h_f = \frac{9 \times l}{59^2 \times 0{\cdot}05} = \frac{9 \times 200}{59^2 \times 0{\cdot}05} = \mathbf{10{\cdot}34\ m} \quad \textbf{Ans.}$$

Example 2 : Frictional loss in pipe running with water is 37·08 m. Diameter and length of the pipe are 15 cm and 3600 m respectively. If rate of flow is 20 l/s then determine coefficient of friction. **(UPBTE)**

Solution : Velocity of flow, $V = \frac{Q}{a} = 20 \times 10^{-3} \times \frac{4}{\pi \times (0{\cdot}15)^2} = 1{\cdot}132$ m/s

Using, $h_f = \frac{4flV^2}{2gd}$

Coefficient of friction, $f = \frac{h_f 2gd}{4lV^2}$

$$\therefore \quad f = \frac{37{\cdot}08 \times 2 \times 9{\cdot}81 \times 0{\cdot}15}{4 \times 3600 \times (1{\cdot}132)^2} = \mathbf{0{\cdot}0059} \quad \textbf{Ans.}$$

Example 3 : A galvanisd iron pipe is discharging 16000 cubic cm water per second. Head loss is not more than 2 m per 100 m length. Determine radius of the pipe if coefficient of friction be 0·005. **(UPBTE)**

Solution : Here $Q = 16000$ cm³/s $= 16 \times 10^{-3}$ m³/s

Let d be the diameter of pipe

Velocity, $V = \frac{Q}{a} = 16 \times 10^{-3} \times \frac{4}{\pi d^2}$

Using, $h_f = \frac{4flV^2}{2gd}$

$$2 = \frac{4 \times 0{\cdot}005 \times 100}{2 \times 9{\cdot}81 \times d} \times \left(\frac{16 \times 10^{-3} \times 4}{\pi d^2}\right)^2$$

$$\therefore \quad d^5 = 2{\cdot}12 \times 10^{-5}$$

$$\therefore \quad \mathbf{d = 0{\cdot}116\ m}, \text{ so radius} = \frac{0{\cdot}116}{2} = \mathbf{0{\cdot}058\ m} \quad \textbf{Ans.}$$

Example 4 : A 50 m long pipe of 300 mm diameter is carrying water at 5 m/s velocity. Determine head loss due to friction. Use Chezy's formula. Take C = 60. **(UPBTE 2002)**

Solution : Flow velocity by Chezy's formula,

$$V = C\sqrt{(mi)}$$

$$i = \frac{h_f}{l} = \frac{h_f}{50}$$

$$m = \frac{A}{P} = \frac{\pi d^2/4}{\pi d} = \frac{d}{4} = \frac{0{\cdot}3}{4} = 0{\cdot}075 \text{ m}$$

$\therefore$ Hence, $5 = 60\sqrt{(0{\cdot}075 \times h_f/50)}$

head loss, $h_f = \frac{5^2}{60^2} \times \frac{50}{0{\cdot}075} = 4{\cdot}629$ m **Ans.**

Example 5 : Diameter of a 400 m long pipe is 30 cm. Water is being discharged at 150 l/s rate. Assuming Darcy's frictional coefficient f = 0·01 determine pressure difference between ends of horizontal pipe. If one end is 4 m above other end then what will be the pressure difference? **(UPBTE)**

Solution : Here $d = 30$ cm $= 0{\cdot}3$ m, $Q = 150$ l/s $= 150 \times 10^{-3}$ m³/s

Velocity of flow, $V = \frac{Q}{A} = \frac{150 \times 10^{-3}}{\frac{\pi}{4} \times 0{\cdot}3^2} = \frac{60}{9\pi} = 21{\cdot}2$ m/s

Head loss, $h_f = \frac{4flV^2}{2gd} = \frac{4 \times 0{\cdot}01 \times 400 \times (2{\cdot}12)^2}{2 \times 9{\cdot}81 \times 0{\cdot}3} = 12{\cdot}23$ m pressure head difference.

$$\frac{p}{\rho g} = \frac{(p_1 - p_2)}{\rho g} = h_f$$

∴ Pressure difference, $p = \rho g \times h_f = 1000 \times 9{\cdot}81 \times 12{\cdot}23 = $ **119·976 kPa** Ans.

Pressure head difference when one end is 4 m above,

$$h = h_f + 4 = 12{\cdot}23 + 4 = 16{\cdot}23 \text{ m}$$

Now, pressure difference, $p = 1000 \times 9{\cdot}81 \times 16{\cdot}23 = $ **159·216 kPa** Ans.

Example 6 : Derive relation $f = \frac{64}{R_e}$ for laminar flow in a normal pipe where f and R_e are coefficient of friction and Reynold's number. **(UPBTE 2009)**

Solution : According to Hagen – Poiseuille equation for laminar flow in circular pipe, the pressure difference between two points of pipe,

$$p_1 - p_2 = \frac{32\mu VL}{d^2} \quad \text{...(i)}$$

According to Darcy – Weisback equation head loss due to friction between two points of a pipe is given by,

$$h_f = \frac{4 flV^2}{2gd}$$

But $h_f = \frac{p_1}{\rho g} - \frac{p_2}{\rho g}$ and assuming $4f = f$,

we have
$$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{flV^2}{2gd}$$

or
$$p_1 - p_2 = \frac{fl\rho V^2}{2d} \quad \text{...(ii)}$$

Equating RHS of eq. (i) and (ii),

$$\frac{fl\rho V^2}{2d} = \frac{32\mu Vl}{d^2}$$

or
$$f = \frac{64\mu}{\rho Vd} = \frac{64}{\rho Vd/\mu}$$

∴
$$f = \frac{64}{R_e} \quad \text{Ans.}$$

Example 7 : A 60 m long pipe of 200 mm diameter is discharging water at 2·5 m/s velocity. Determine head loss due to friction. **(UPBTE 2011)**

Solution : Here $l = 60$ m, $d = 200$ mm $= 0{\cdot}2$ m, $V = 2{\cdot}5$ m/s

Frictional coefficient, $f = 0{\cdot}005\left(1+\frac{1}{35d}\right)$ (New pipe)

and $f = 0{\cdot}01\left(1+\frac{1}{35d}\right)$ (Old pipe)

For new pipe : $f = 0{\cdot}005\left(1+\frac{1}{35\times0{\cdot}2}\right) = 0{\cdot}0057$

Now head loss,
$$h_f = \frac{4flV^2}{2gd}$$
$$= \frac{4\times0{\cdot}0057\times60\times(2{\cdot}5)^2}{2\times9{\cdot}81\times0{\cdot}2} = 2{\cdot}18 \text{ m} \quad \text{Ans.}$$

For old pipe : $f = 0{\cdot}01\left(1+\frac{1}{35\times0{\cdot}2}\right) = 0{\cdot}011$

Now head loss,
$$h_f = \frac{4\times0{\cdot}011\times60\times(2{\cdot}5)^2}{2\times9{\cdot}81\times0{\cdot}2} = 4{\cdot}20 \text{ m} \quad \text{Ans.}$$

8·6. Energy or Head Losses of Flowing Liquid

When a liquid, flowing through a pipe, suffers change in velocity either in magnitude or direction, a turbulence is created due to formation of eddies. Some part of the energy possessed by liquid is converted into heat. This is termed as ***'loss of energy or head'***.

Since loss of various types of energy vary with square of mean velocity therefore, losses are expressed in terms of 'velocity head'.

All types of energy losses are classified as follows—

1. **Major energy loss**—Head loss due to friction is major energy loss. Darcy – Weisback formula is used to determine major energy loss.
2. **Minor energy losses**—These are caused by change in velocity.
   (i) Head loss due to sudden enlargement of pipe
   (ii) Head loss due to sudden contraction of pipe
   (iii) Head loss on entry to a pipe from vessel
   (iv) Head loss due to obstruction in pipe
   (v) Head loss due to bend in pipe
   (vi) Head loss due to gradual contraction or enlargement of pipe
   (vii) Head loss at exit from a pipe.

8·6·1. Head Loss Due To Sudden Enlargement Of Pipe

Consider a pipe as shown in fig. 8·4. Cross-sectional area of the pipe suddenly expands from a_1 to a_2. Due to sudden enlargement liquid suffers turbulence and eddies are formed near corners of expansion. Head loss due to sudden enlargement is due to formation of eddies.

Let V_1, a_1, p_1 = velocity, cross-sectional area and pressure intensity at section 1 – 1

only formulae

V_2, a_2, p_2 = velocity, cross-sectional area and pressure intensity at section – 2 –2

$(a_2 - a_1)$ = area of eddies

p = pressure intensity at eddies

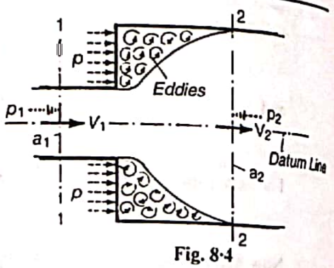


Fig. 8·4

Now concentrate on liquid contained within sections 1 – 1 and 2 – 2.

Net force on this liquid,

$$= p_2 a_2 - p_1 a_1 - p(a_2 - a_1)$$

but by experiment it is found that

$$p = p_1$$

$\therefore$ Net force on liquid $= p_2 a_2 - p_1 a_1 - p_1(a_2 - a_1)$

$$= a_2 (p_2 - p_1)$$

Let W be the weight of the liquid flowing per second. so. from

$$W = \rho g a_1 V_1 = \rho g a_2 V_2 \quad \text{(where } \rho = \text{density of liquid)}$$

As flow velocity changes from V_1 to V_2,

So, momentum of liquid at section 1 – 1 $= \frac{W}{g} \cdot V_1$

Momentum of liquid at section 2 – 2 $= \frac{W}{g} \cdot V_2$

Thus, rate of change of momentum $= \frac{W}{g} \cdot V_1 - \frac{W}{g} \cdot V_2$

$$= \frac{\rho g a_2 V_2 V_1}{g} - \frac{\rho g a_2 V_2 V_2}{g}$$

Force on liquid between two sections = Rate of change in momentum

$$\therefore \quad a_2(p_2 - p_1) = \frac{\rho g a_2 V_2 V_1}{g} - \frac{\rho g a_2 V_2^2}{g}$$

or

$$\frac{(p_2 - p_1)}{\rho g} = \frac{V_2 V_1}{g} - \frac{V_2^2}{g} \qquad ...(1)$$

Suppose h_{EL} be the head loss due to sudden enlargement. Now applying Bernoulli's theorem at section 1 – 1 and 2 – 2,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_{EL}$$

assuming centre line of pipe as datum line,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_{EL}$$

$$\therefore \quad h_{EL} = \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - \frac{(p_2 - p_1)}{\rho g}$$

From relation (1),

$$h_{EL} = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - \left(\frac{V_2 V_1}{g} - \frac{V_2^2}{g}\right)$$

$$= \frac{V_1^2}{2g} + \frac{V_2^2}{2g} - \frac{V_2 V_1}{g} = \frac{(V_1 - V_2)^2}{2g}$$

hence head loss,

$$\boxed{h_{EL} = \frac{(V_1 - V_2)^2}{2g}} \qquad ...(2)$$

This expression was obtained by two scientists therefore known as Borda – Carnot equation.

Discussion—*Sudden enlargement in the pipe should always be avoided. For example, area at section 2 – 2 is four times the area of section 1 – 1, i.e. $a_2 = 4a_1$*

so, from continuity theorem, $a_1 V_1 = a_2 V_2$

or

$$V_2 = \frac{V_1}{4}$$

$\therefore$ *head loss due to sudden enlargement,*

$$h_{EL} = \frac{(V_1 - V_1/4)^2}{2g} = \frac{9}{16}\frac{V_1^2}{2g}$$

Therefore, more than half of velocity head at section 1 – 1 is lost due to sudden enlargement.

8·6·2. Head Loss Due to Sudden Contraction Of Pipe

Consider a pipe as shown in fig. 8·5. Cross-section area of the pipe suddenly reduces from a_1 to a.

It can be seen that cross-section of flow reduces from a_1 to a_c at vena – contracta. Flow area thereafter increases to a.

Now focus on liquid between section $C – C$ and section 2 – 2. It can be understood that loss of head is actually due to sudden enlargement between vena – contracta and smaller pipe.

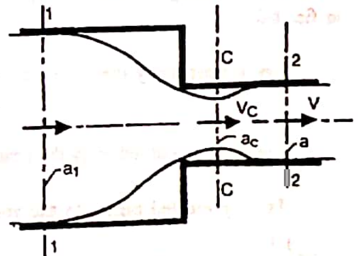


Fig. 8·5

Therefore, head loss between $C – C$ and 2 – 2,

$$h_c = \frac{(V_c - V)^2}{2g}$$

Using continuity equation,

$$a_c V_c = aV$$

But

$$\frac{a_c}{a} = C_c = \text{coefficient of contraction}$$

$$\therefore \quad V_c = \frac{V}{a_c / a} = \frac{V}{C_c}$$

therefore,
$$h_c = \frac{\left(\frac{V}{C_c} - V\right)^2}{2g}$$

Hence, head loss due to sudden contraction,

$$h_c = \left(\frac{1}{C_c} - 1\right)^2 \frac{V^2}{2g}$$

Substituting $C_c = 0{\cdot}62$ in above equation,

head loss,
$$h_c = \left(\frac{1}{0{\cdot}62} - 1\right)^2 \frac{V^2}{2g}$$

or
$$h_c = 0{\cdot}375 \frac{V^2}{2g}$$

But in practice head loss due to sudden contraction,

$$\boxed{h_c = 0{\cdot}5 \frac{V^2}{2g}}$$

8·6·3. Head Loss On Entry To A Pipe From a Vessel

When liquid enters into a pipe from a tank or reservoir some loss of head occurs at entry to the pipe. The flow pattern of liquid is similar to the head loss due to sudden contraction as shown in fig. 8·5.

Loss of energy on entry to a pipe depends on type of entry as shown in fig. 8·6.

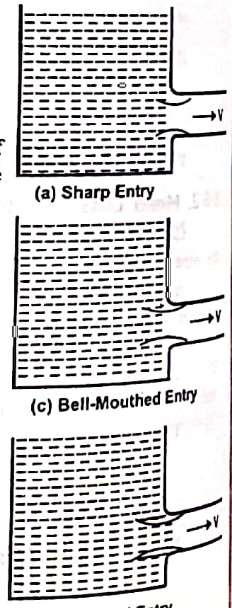
For a sharp entry the head loss at entrance is taken, $0{\cdot}5 \frac{V^2}{2g}$

For a bell–mouthed entry the head loss at entrance is taken, $0{\cdot}05 \frac{V^2}{2g}$

For a projected pipe into the vessel, head loss at entrance is taken, $0{\cdot}78 \frac{V^2}{2g}$ where V is velocity of flow in the pipe.

However, an entry into the pipe is assumed sharp and head loss,

$$\boxed{h_{EN} = 0{\cdot}5 \frac{V^2}{2g}}$$



(a) Sharp Entry

(c) Bell-Mouthed Entry

(c) Projected Entry

Fig. 8·6

8·6·4. Head Loss Due To Obstruction In Pipe

Consider an obstruction in the pipe as shown in fig. 8·7. Such destructions are usually due to projection of gasket, flange or valve etc. When valve is partially opened cross-section of pipe contracts first and followed by sudden enlargement immediately.

Let a = area of cross-section of pipe
a_o = maximum area of obstruction
V = velocity flow in the pipe
V_c = velocity of flow at section $C - C$ of obstruction

Now applying continuity equation,

$$aV = (a - a_0)\, V_c \times C_c$$

or
$$V_c = \frac{a.V}{(a - a_0)C_c}$$

where C_c = coefficient of contraction

Head loss due to obstruction which is same as head loss due to sudden enlargement,

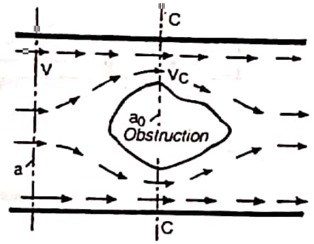


Fig. 8·7

$$h_{OB} = \frac{(V_c - V)^2}{2g}$$

or
$$h_{OB} = \frac{\left[\frac{a.V}{(a - a_0)C_c} - V\right]^2}{2g}$$

$$\therefore \quad h_{OB} = \left[\frac{a}{(a - a_0)C_c} - 1\right]^2 \frac{V^2}{2g}$$

Average value of C_c is taken as 0·65 which should be used to calculate K.

If
$$\left[\frac{a}{(a - a_0)C_c} - 1\right]^2 = K$$

head loss,
$$\boxed{h_{OB} = K \frac{V^2}{2g}}$$

8·6·5. Head Loss Due To Bend In Pipe

Bends are provided in a flow passage to change the direction of flow. Change in direction of flow also adds to the loss of head.

Path of liquid as shown in fig. 8·8. It should be noted that in bends, elbows, Tee – pieces the eddies are formed same way as in case of sudden enlargement. Like any other minor loss, the head loss due to bend is a function of $\frac{V^2}{2g}$, where V is the mean velocity of flow.

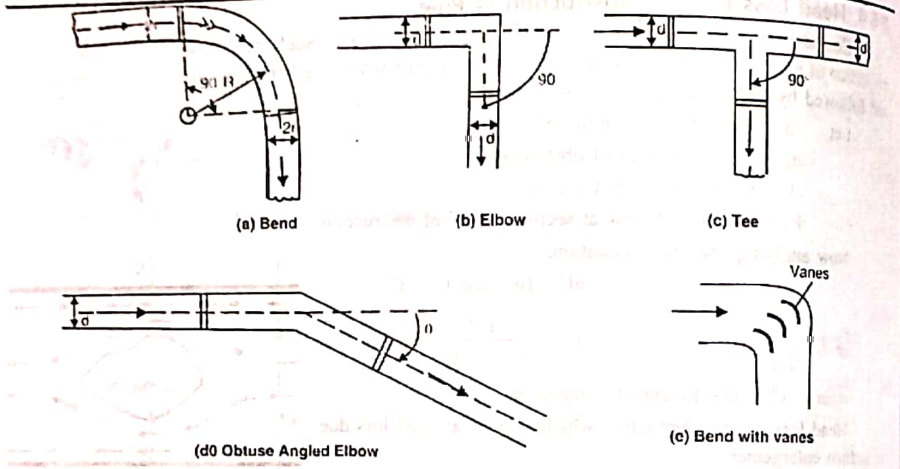


Fig. 8·8

(a) Head loss due to bend, $$h_B = K\frac{V^2}{2g}$$

Where value of K = 0·2 to 0·3. However, average value of K is taken as 0·25 for the ratio of radius of curvature to the radius of pipe R/r greater than 5.

(b) and (c) Head loss due to right–angled elbow or Tee piece.

$$h_B = \sin^2 90° \frac{V^2}{2g}$$

or

$$h_B = \frac{V^2}{2g}$$

(c) Head loss due to obtuse – angled elbow,

$$h_B = \sin^2\theta\frac{V^2}{2g}$$

Above losses may be reduced by using a grid of deflecting vanes across the cross-section of bend as shown in fig. 8·8 (e). Value of K then, is not more than 0·15.

8·6·6. Head Loss Due To Gradual Contraction or Enlargement Of Pipe

Loss of energy of flowing liquid can be minimised by employing gradual contraction or enlargement of pipe in place of sudden contraction or enlargement.

Reason is obvious. Formation of eddies responsible for head loss is eliminated in case of gradual pipe.

For gradual enlargement, for example divergence cone of venturimeter or draft tube of turbine, head loss is given by,

$$h_{GE} = K\frac{(V_1 - V_2)^2}{2g}$$

Where K is a constant whose value is less than one. Value of K depends upon (i) angle of divergence and (ii) ratio of diameters at two ends of gradual pipe. V_1 and V_2 be mean velocity of flow at two gradual ends.

For gradual contraction the value of K is very small. Therefore, for a gradual contraction without sharp corner loss of energy can be neglected. But sometimes following formula is used.

$$h_{GC} = 0{\cdot}05\frac{V^2}{2g}$$

Where V is mean velocity of flow at small end of gradual contraction.

8·6·7. Head Loss At Exit From A Pipe

When a pipe discharges into a large reservior or into atmosphere, the kinetic energy available at the end of pipe is lost. If V be the velocity of flow at the end of pipe then head loss due at the exit,

$$h_{EX} = \frac{V^2}{2g}$$

Exit end from a pipe is assumed of infinite area of cross-section $A_2 = \infty$, which results $V_2 = 0$. Thus, applying formula of head loss due to sudden enlargement,

$$h_{EX} = \frac{(V_1 - V_2)^2}{2g}$$

here $V_1 = V$ and $V_2 = 0$,

$$\therefore \qquad h_{EX} = \frac{V^2}{2g}$$

8·7. Hydraulic Gradient line

Consider fig. 8·9. Two tanks are joined by a pipe of uniform cross-section. Liquid flows from tank – 1 to tank – 2. Piezometer tubes are fitted at various points of the tube. Height of liquid in piezometer tube shall give pressure heads $\frac{p}{\rho g}$ at these points. Join liquid surface of each piezometer tube. We obtain a straight sloping line, which is known as hydraulic gradient line (HGL), piezometric line, hydraulic grade line or pressure grade line.

If pipe is very long, then minor head losses will be negligible as compared to head loss due to friction. Then, hydraulic gradient line shall pass through the liquid surface of both tanks as shown.

Now assume a datum line passing through bottom of tank – 2. Height of centre line of pipe Z at any point is static head of the liquid. *Sum of static head and pressure head* $\left(Z + \frac{p}{\rho g}\right)$ *at any point of the pipe*

But $(H + Z) - (H' + Z')$ = difference in height of liquid surfaces in tanks $= h$

$$\therefore \quad h = \frac{0 \cdot 5 V_1^2}{2g} + \frac{4 f l_1 V_1^2}{2 g d_1} + \frac{0 \cdot 5 V_2^2}{2g} + \frac{4 f l_2 V_2^2}{2 g d_2} + \frac{(V_2 - V_3)^2}{2g} + \frac{4 f l_3 V_3^2}{2 g d_3} + \frac{V_3^2}{2g}$$

8·9. Discharge Through Pipe Into Atmosphere

Consider a pipe line of uniform cross-section connected to tank as shown in fig. 8·11. Pipe discharges liquid into atmosphere.

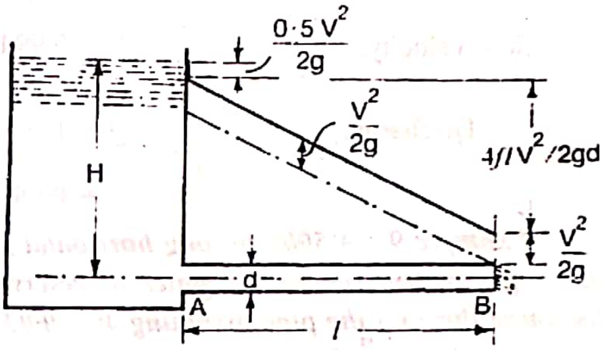


Fig. 8·11

Let d = diameter of the pipe

l = length of the pipe

V = flow velocity

f = coefficient of friction

Then, $0 \cdot 5 \dfrac{V^2}{2g} = h_{EN}$ = head loss at entry point A.

$\dfrac{4 f l V^2}{2 g d} = h_f$ = head loss due to friction in pipe AB.

$\dfrac{V^2}{2g} = h_{EX}$ = head loss at exit point B

Now applying Bernoulli's theorem at point A and B,

$$\boxed{H = 0 \cdot 5 \frac{V^2}{2g} + \frac{4 f l V^2}{2 g d} + \frac{V^2}{2g}}$$

Note—1. This equation can be used to determine flow velocity.

2. Once V is known discharge of liquid can be determined as diameter is also known.

3. Vertical displacement between TEL and HGL is $\dfrac{V^2}{2g}$.

Example 8 : A 196 m long pipe of 100 mm diameter is connected to a reservior. Pipe discharges water in atmosphere. Head of water is 4 m in the reservior. Determine discharge of water. Take f = 0·01 and neglect head loss at entry and exit. ***(UPBTE)***

Solution : Applying Bernoulli's theorem at entry and exit of the pipe,

$$H = Z + \frac{V^2}{2g} + h_f$$

Let centre line of pipe is assumed as datum line.

then $Z = 0$, now

$$4 = \frac{V^2}{2g}\left[1 + \frac{4fl}{d}\right]$$

$$4 \times 2 \times 9{\cdot}81 = V^2\left[1 + \frac{4 \times 0{\cdot}01 \times 196}{0{\cdot}1}\right]$$

flow velocity, $V = 0{\cdot}994$ m/s

∴ Discharge, $Q = V \times a = 0{\cdot}994 \times \frac{\pi}{4} \times (0{\cdot}1)^2$

$= 0{\cdot}0078$ **m³/s** Ans.

Example 9 : A 5000 m long horizontal pipe of diameter 80 cm is connected to a reservior which discharges water. Surface of water of reservior is 200 m above the centre line of pipe. Determine discharge through the pipe assuming $4f = 0{\cdot}02$ and neglecting minor energy losses. Determine pressure head at mid point of pipe also. **(UPBTE)**

Solution : Here $l = 5000$ m, $d = 80$ cm $= 0{\cdot}8$ m and $4f = 0{\cdot}02$

Head loss due to friction, $h_f = \frac{4flV^2}{2gd} = \frac{0{\cdot}02 \times 5000 \times V^2}{2 \times 9{\cdot}81 \times 0{\cdot}8}$

or $h_f = 6{\cdot}371\ V^2$

where V is velocity of water.

If pipe discharges water into atmosphere then head loss at exit

$$= \frac{V^2}{2g}$$

Now, applying Bernoulli's theorem between ends,

$$H = h_f + \frac{V^2}{2g}$$

or $200 = 6{\cdot}371\ V^2 + \frac{V^2}{2g}$

∴ $V = 5{\cdot}58$ m/s

Cross-sectional area of pipe, $a = \frac{\pi}{4} \times (0{\cdot}8)^2 = 0{\cdot}502$ m²

hence discharge, $Q = aV = 0{\cdot}502 \times 5{\cdot}58 =$ **2·801 m³/s** Ans.

Head loss due to friction upto mid of the length,

$$h_{f1} = \frac{4f(l/2)V^2}{2gd} = \frac{0{\cdot}02(5000/2) \times (5{\cdot}58)^2}{2 \times 9{\cdot}81 \times 0{\cdot}8}$$

$= 99{\cdot}18$ **m** Ans.

Now, applying Bernoulli's theorem between entry and mid point of the pipe,

$$H = h_{f1} + \frac{p}{\rho g} + \frac{V^2}{2g}$$

pressure head at mid point of the pipe,

$$\frac{p}{\rho g} = H - h_{f1} - \frac{V^2}{2g}$$

$$= 200 - 99{\cdot}18 - \frac{(5{\cdot}58)^2}{2 \times 9{\cdot}81}$$

$= 99{\cdot}234$ **m** Ans.

Example 10 : A 5 km long pipe line of diameter 0·3 m discharges water into atmosphere. Other end of the pipe is connected to a reservior. Water flows at 1 m/s velocity through pipe. Determine total head loss in the pipe. Take $4f = 0{\cdot}04$ and assume sharp entry from reservior to pipe. Determine discharge of water also. **(UPBTE)**

Solution : Let V be the velocity of flow. Head loss at entry in pipe from reservior,

$$= \frac{0{\cdot}5V^2}{2g}$$

Head loss due to friction $= \frac{4f.l.V^2}{2gd}$

Head loss at exit from pipe $= \frac{V^2}{2g}$

Total head loss $= \frac{0{\cdot}5V^2}{2g} + \frac{4flV^2}{2gd} + \frac{V^2}{2g}$

$$= \frac{V^2}{2g}\left[0{\cdot}5 + \frac{4fl}{d} + 1\right] = \frac{1^2}{2 \times 9{\cdot}81}\left[0{\cdot}5 + \frac{0{\cdot}04 \times 5 \times 10^3}{0{\cdot}3} + 1\right]$$

$=$ **34·1 m water column** Ans.

Cross-section area of the pipe, $a = \frac{\pi}{4} \times (0{\cdot}3)^2 = 0{\cdot}0706$ m²

∴ Discharge, $Q = aV = 0{\cdot}0706 \times 1 =$ **0·0706 m³/s** Ans.

Therefore, power transmission through a pipe is maximum when head loss due to friction is one-third of total head.

Maximum efficiency of supplied power

Putting $h_f = \frac{H}{3}$ in efficiency

$$\eta_{max.} = \frac{H - H/3}{H} = \frac{2}{3} = 66{\cdot}67\%$$

8·13. Water Hammer

A flow through a pipe possess momentum (MV) due to mass M and velocity V. If water flowing through pipe is suddenly brought to rest by closing valve or by other means, the momentum of water reduces to zero instantaneously. It causes generation of high pressure waves which travel along the pipe. High pressure wave create noise known as ***knocking***. Therefore, the phenomenon of sudden rise in pressure due to sudden closure of flow is known as ***water hammer*** or ***hammer blow***.

Such high pressure may even cause bursting of pipe. Therefore, water hammer must be considered while designing pipe. Generation of high pressure depends upon velocity of flow, speed of closure, length of the pipe, density of liquid and elastic properties of pipe material.

Detailed study of water hammer is beyond the scope of this book. However, we shall limit to the study of gradual closure of pipe.

Let
a = cross-sectional area of the pipe
l = length of the pipe
ρ = density of the liquid
V = velocity of flow of the liquid
t = time taken to close the valve

Mass of liquid, $$M = \frac{\rho g.a.l}{g}$$

Rate of decrease of velocity

$$\frac{V - 0}{t} = \frac{V}{t}$$

But force, $$F = \text{Mass} \times \text{Rate of change of velocity}$$

∴ Force $$F = \frac{\rho g.a.l}{g} \times \frac{V}{t}$$

∴ Pressure intensity, $$p = \frac{F}{a} = \frac{\rho g.a.l}{g} \times \frac{V}{t} \times \frac{1}{a} = \frac{\rho g l V}{gt}$$

hence $$p = \frac{\rho l V}{t}$$

Example 29 : ***Determine power of head loss due to friction of a pipe of circular cross-section. Pipe is 300 m long, 15 cm diameter and rate of flow is 28 l/s through the pipe. Take f = 0·01.***

(UPBTE, Rajasthan)

Solution : Discharge, $Q = 28\ l/s = 28 \times 10^{-3}\ m^3/s$